

Non-Gaussian Inflationary Perturbations from the dS/CFT Correspondence

David Seery and James E. Lidsey

Astronomy Unit, School of Mathematical Sciences
Queen Mary, University of London
Mile End Road, London E1 4NS
United Kingdom

E-mail: D.Seery@qmul.ac.uk, J.E.Lidsey@qmul.ac.uk

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Abstract. We use the dS/CFT correspondence and bulk gravity to predict the form of the renormalized holographic three-point correlation function of the operator which is dual to the inflaton field perturbation during single-field, slow-roll inflation. Using Maldacena's formulation of the correspondence, this correlator can be related to the three-point function of the curvature perturbation generated during single-field inflation, and we find exact agreement with previous bulk QFT calculations. This provides a consistency check on existing derivations of the non-Gaussianity from single-field inflation and also yields insight into the nature of the dS/CFT correspondence. As a result of our calculation, we obtain the properly renormalized dS/CFT one-point function, including boundary contributions where derivative interactions are present in the bulk. In principle, our method may be employed to derive the n -point correlators of the inflationary curvature perturbation within the context of $(n-1)^{\text{th}}$ -order perturbation theory, rather than n^{th} -order theory as in conventional approaches.

Keywords: Inflation, Cosmological perturbation theory, Physics of the early universe, String theory and cosmology

1. Introduction

There has been considerable interest recently in understanding the nature of non-Gaussian features in the primordial curvature perturbation that is generated during early universe inflation [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19]. This is motivated in part by the ever-increasing sensitivity of Cosmic Microwave Background (CMB) data, through which one might hope to detect, or at least set strong upper limits on, the primordial non-Gaussianity [20, 21, 22, 23, 24, 25]. Moreover, from a theoretical perspective, the form of the primordial three-point correlation function may provide a more sensitive discriminant of inflationary microphysics than the tilt of the perturbation spectrum [26, 27, 3, 28, 29, 30].

Given these considerations, there is a pressing need to develop accurate theoretical techniques for calculating the primordial three-point function in concrete inflationary scenarios. An elegant method for determining the level of non-Gaussianity at horizon crossing in standard single-field, slow-roll inflation has been developed by Maldacena [2]. In this approach, the tree-level Feynman diagrams for an appropriate vacuum-to-vacuum expectation value are evaluated. This technique was subsequently applied by various authors to other inflationary scenarios, including models where higher-derivative operators are present in the inflationary Lagrangian, or where more than one field is dynamically important [3, 4, 5, 26, 27]. It has been further demonstrated that the calculations can be extended beyond tree-level to include the effect of loop corrections [31].

The purpose of the present paper is to develop an alternative method for calculating the three-point correlator for the inflaton field perturbation that is based on the conjectured de Sitter/Conformal Field Theory (dS/CFT) correspondence [32, 33, 2]. This correspondence states that quantum gravity in four-dimensional de Sitter space is dual to a three-dimensional Euclidean CFT. In this picture, the timelike coordinate in de Sitter space is viewed as the scale parameter of the CFT and slow-roll inflation may be then interpreted as a deformation of the CFT away from perfect scale invariance [32, 34, 35]. It is natural, therefore, to investigate the relationship between the perturbations that are generated in the bulk inflationary physics and those of the holographically dual boundary field theory. The general rules for computing correlation functions in the dS/CFT framework were presented by Maldacena [2] and shown to produce the correct results in the case of a massless scalar field. The massive case was considered in [36], where the β -function and anomalous dimension in the dual CFT were related to the inflationary slow-roll parameters ϵ and η , which measure the logarithmic slope and curvature of the inflaton potential, respectively. The renormalized CFT generating functional that is dual to Einstein gravity coupled to a scalar field was calculated by Larsen & McNeen [37] (see also [38]), who demonstrated that it correctly reproduces the amplitude and spectral tilt of the density perturbation spectrum derived from standard bulk quantum field theory (QFT) calculations. (For reviews of such calculations, see, e.g., [39, 40]).

The proposal that quantum gravity in de Sitter space is holographically related to a CFT in one dimension fewer is motivated by the analogous case of the AdS/CFT correspondence. A concrete realization of this correspondence is provided by perturbative type IIB string theory on $\text{AdS}_5 \times \mathbf{S}^5$ with N branes in the near-horizon limit. This can be related to an $\mathcal{N} = 4$, $SU(N)$ super-Yang Mills theory on the boundary of AdS [41, 42, 43] (where $N \gg 1$; for more details, see, e.g., [44]). It is also possible to go beyond this perturbative construction to more general scenarios. Indeed, it has been suggested that the AdS/CFT correspondence may be viewed as a definition of what is meant by quantum gravity in a space of asymptotically constant negative curvature [45].

However, the situation for the dS/CFT correspondence is less clear. There is presently no known concrete framework available which relates perturbative string theory on dS space to some conformal field theory on the dS boundary. Indeed, there may even be reasons to believe that such a correspondence does not actually exist [45, 46, 47, 48], at least for generic values of the Hubble rate H and Newton's constant G . Although the dS and AdS manifolds are related by a double Wick rotation of the timelike and radial coordinates, this property does not carry over to the construction of holographic partners. We will encounter some of the consequences of this in the analysis outlined below.

We will work with the more general version of the dS/CFT correspondence, which states that *any* gravitational theory in an asymptotically dS space is holographically dual to some CFT on the boundary. In this case, one can obtain information about the CFT by working entirely from the bulk theory and the assumed dS asymptotics. This is the approach taken in much of the AdS/CFT literature [2, 32, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60] and in previous applications of the dS/CFT correspondence to cosmology [36, 38, 37]. Given that the precise form of the dS/CFT correspondence is at present unknown, we will compute the form that the holographic correlation functions should take if the bulk calculation is to be recovered. If the holographic CFT that is dual to bulk inflation is identified in the future, a direct calculation of its correlation functions will then be possible and this will allow a comparison to be made with the results of the present work. More specifically, we use Maldacena's formulation of the dS/CFT correspondence [2] to explicitly calculate the bulk prediction for the three-point correlator of the dual CFT which reproduces the primordial three-point correlator of the inflaton field perturbation in single-field, slow-roll inflation. This provides a valuable consistency check of pre-existing results and yields further insight into the nature of the dS/CFT correspondence itself and, more generally, into the properties of quantum gravity in de Sitter spacetime.

The paper is organised as follows. We begin in Section 2 by outlining those features of the dS/CFT correspondence that will be relevant for what follows and proceed in Section 3 to present the bulk-to-boundary and bulk-to-bulk propagators for both free and interacting fields. This, together with the apparatus of holographic renormalization [61, 51, 53], provides the necessary machinery for calculating the holographic one-point

function $\langle \mathcal{O} \rangle$ of the operator \mathcal{O} which is dual to the inflaton φ . The derivation of $\langle \mathcal{O} \rangle$ is performed in Section 4. The argument is similar to that adopted in the AdS/CFT framework [50, 49, 53, 51], but an extra subtlety arises due to the presence of derivative interactions in the bulk. These interactions cause the appearance of non-zero boundary terms at future infinity which can not necessarily be discarded, and this has important consequences when calculating the holographic one-point function.

Once the form of the one-point function has been determined, all other connected correlation functions can be derived directly from it and we obtain the two- and three-point functions in Section 5. Up to this point in the discussion, the analysis has been kept general, in the sense that the specific form of the interaction vertex has not been specified. In Section 6, we discuss the effective field theory for the inflaton field during inflation, paying particular attention to the role of boundary terms in the action, and derive the corresponding holographic two- and three-point functions. We then proceed in Section 7 to demonstrate explicitly that the dS/CFT approach correctly leads to Maldacena's bulk QFT result for the primordial three-point non-Gaussianity in single-field inflation [2]. We also emphasize that the boundary terms that appear in the third-order bulk action for the inflaton perturbation can be accounted for by either performing a suitable field redefinition or by including such terms explicitly. Finally, we conclude with a discussion in Section 8.

2. The dS/CFT Correspondence

In this section we briefly review the dS/CFT [32, 48, 62, 63] conjecture, in the variant proposed by Maldacena [2]. The metric for the dS spacetime can be expressed in the form

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j = a^2(\eta)[-d\eta^2 + \delta_{ij}dx^i dx^j], \quad (1)$$

where t denotes cosmic time, $\eta = \int dt/a(t)$ defines conformal time and the scale factor $a = e^{Ht} = -(H\eta)^{-1}$. In these coordinates, slices of constant t are manifestly invariant under the 3-dimensional Riemannian Poincaré group, $ISO(3)$. The far past ($t \rightarrow -\infty$) corresponds to $\eta \rightarrow -\infty$ and the far future ($t \rightarrow \infty$) to $\eta \rightarrow 0^-$. The boundary of dS space, ∂dS , lies at $t = +\infty$, together with a point at $t = -\infty$, which makes the boundary compact [42]. The dual CFT can loosely be thought of as living on the Riemannian slice at $\eta \rightarrow 0^-$, and inherits its symmetries.

The symmetries of dS space are broken in the presence of a scalar field, ϕ , which propagates according to the field equation

$$\ddot{\phi} + 3H\dot{\phi} + \left(\frac{k^2}{a^2} + m^2\right)\phi = -\frac{dV(\phi)}{d\phi}, \quad (2)$$

where we have translated to Fourier space, m denotes the mass of the field and possible higher-order interactions are parametrized by a potential $V(\phi)$. Even if ϕ evolves homogeneously, the scale factor will be deformed owing to the presence of energy-momentum in the bulk. Nonetheless, if the homogeneous part of the scalar

field dominates as $t \rightarrow \infty$, the metric will be asymptotically de Sitter such that $a(t) \sim e^{Ht}[1 + O(t^{-1})]$. In this case, provided ϕ is sufficiently light ‡ , it will behave near the boundary as

$$\phi \sim \hat{\phi} e^{\Delta_+ Ht} (1 + \dots) + \bar{\phi} e^{\Delta_- Ht} (1 + \dots), \quad (3)$$

where

$$\Delta_\pm = -\frac{3}{2} \pm \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}. \quad (4)$$

The boundary conditions determine the constants of integration $\{\hat{\phi}, \bar{\phi}\}$. It follows that

$$\phi \sim \hat{\phi} e^{\Delta_+ Ht} \quad (5)$$

on the approach to ∂dS (as $t \rightarrow \infty$). Note that $\phi \rightarrow \hat{\phi}$ in the massless limit $m \rightarrow 0$.

Analogous to the AdS/CFT correspondence [41, 42, 64, 43, 60], the dS/CFT correspondence is the formal statement that the wavefunction of quantum gravity on de Sitter space, Ψ_{dS} , is given by the partition function of a dual CFT [32, 2]:

$$Z_{\text{CFT}}[\hat{\phi}] = \Psi_{\text{dS}} \approx e^{iS_{\text{cl}}[\hat{\phi}]}, \quad (6)$$

where the second (approximate) equality holds when the curvature in the four-dimensional bulk spacetime is sufficiently small that the path integral arising in the definition of the wavefunction can be evaluated in the semi-classical limit. This yields the on-shell bulk action $S_{\text{cl}}[\hat{\phi}]$ evaluated on the classical solution. This action is a functional of the boundary data $\hat{\phi}$.

Eq. (6) implies that the generating function of the CFT, $Z_{\text{CFT}}[\hat{\phi}]$, must also depend on $\hat{\phi}$. However, a generating function is typically only a functional of the sources in the theory, since all dynamical fields are integrated out. Therefore, it is natural to interpret $\hat{\phi}$ as the source for some operator \mathcal{O} which is dual, up to a constant of proportionality, to ϕ under the dS/CFT correspondence. This implies that Eq. (6) can be expressed as [41, 42, 50, 60, 65, 66]

$$\left\langle \exp \left(\int_{\partial\text{dS}} d^3x \hat{\phi} \mathcal{O} \right) \right\rangle_{\text{CFT}} = Z_{\text{CFT}}[\hat{\phi}] \approx \exp \left(iS_{\text{cl}}[\hat{\phi}] \right), \quad (7)$$

where d^3x is an invariant volume measure on the boundary ∂dS . Eq. (7) is the statement of the dS/CFT correspondence that we employ in this paper, although in practice it must be supplemented with counterterms, as discussed in Section 4. The CFT correlators in the absence of the source $\hat{\phi}$ can be recovered by functionally differentiating Eq. (7) with respect to the source, $\hat{\phi}$, and setting $\hat{\phi} = 0$ after the differentiation. Thus,

$$\langle \mathcal{O}(\mathbf{x}_1) \cdots \mathcal{O}(\mathbf{x}_n) \rangle = \left. \frac{\delta^n \ln Z_{\text{CFT}}[\hat{\phi}]}{\delta \hat{\phi}(\mathbf{x}_1) \cdots \delta \hat{\phi}(\mathbf{x}_n)} \right|_{\hat{\phi}=0}. \quad (8)$$

‡ If $m > 3H/4$, the conformal weight in Eq. (4) becomes imaginary. This is one of the difficulties with the dS/CFT proposal. As in Ref. [2], we restrict our attention to light fields, where this problem does not arise. In any event, only fields satisfying $m < 3H/2$ are excited during a de Sitter epoch and could generate a significant curvature perturbation in the late universe.

A physical interpretation may also be given to the other constant $\bar{\phi}$ that arises in the asymptotic solution (3). In the AdS/CFT correspondence, this parameter is identified, modulo a constant of proportionality, as the vacuum expectation value (VEV) $\langle \mathcal{O} \rangle$ [42, 49, 56]. In general, the numerical value of the constant of proportionality depends on the specific theory under investigation and must be evaluated by direct calculation. Since $\hat{\phi}$ is interpreted as a source, the quantity $\bar{\phi}$ is often referred to as the *response* in the AdS/CFT literature. We will show that in the dS/CFT correspondence, the identification $\langle \mathcal{O} \rangle \propto \bar{\phi}$ must be modified by including boundary terms that arise from interactions in the bulk field. We will also explicitly determine the constant of proportionality in Section 4.1.

The interpretation of $\hat{\phi}$ as a source for \mathcal{O} and $\bar{\phi}$ as its VEV has an analogue in the bulk theory [67, 59]. This serves to make the identifications more transparent. The Δ_- solution in Eq. (3) decays near the boundary and is normalizable. It corresponds to a finite energy excitation of the bulk theory, just as the acquisition of a VEV by \mathcal{O} is a finite energy excitation of the boundary CFT. The Δ_+ solution, on the other hand, is not normalizable and corresponds to an infinite energy excitation of the bulk theory. It should therefore be viewed as a deformation of the gravitational background. This is equivalent to the deformation of the CFT Lagrangian by the operator $\hat{\phi}\mathcal{O}$.

3. Bulk-to-Boundary and Bulk-to-Bulk Propagators

The above discussion implies that a key feature of employing the dS/CFT correspondence is the Dirichlet problem in de Sitter space, i.e., the problem of finding the solution to the bulk field equation (2) subject to the boundary condition that $\phi \rightarrow \hat{\phi}$ (for the massless case) in the far future. In this section, we identify the solution that satisfies this property. This will enable us to calculate the response $\bar{\phi}$ in terms of the source $\hat{\phi}$.

If the metric is asymptotically dS as $t \rightarrow \infty$, the self-interaction described by the potential $V(\phi)$ in Eq. (2) becomes unimportant. In this limit, the general solution to Eq. (2) is given in terms of Bessel functions:

$$\phi = (-k\eta)^{3/2} J_\nu(-k\eta) \bar{\phi} + (-k\eta)^{3/2} Y_\nu(-k\eta) \hat{\phi}, \quad (9)$$

where [68, 69, 70]

$$\nu^2 = \frac{9}{4} - \frac{m^2}{H^2} = \left(\Delta_\pm + \frac{3}{2} \right)^2. \quad (10)$$

One of the boundary conditions that can be imposed is that the field should be in its vacuum state in the asymptotic past when it is deep inside the de Sitter horizon. The usual choice is to invoke the Bunch–Davies vacuum [71], which corresponds to specifying the solution in terms of a Hankel function of the second kind or order ν :

$$\phi \propto (-k\eta)^{3/2} H_\nu^{(2)}(-k\eta). \quad (11)$$

In the following Subsections, we determine the constant of proportionality in Eq. (11) for free and interacting fields, respectively.

3.1. Free Fields

A free field has $m = V = 0$. As discussed above, the second boundary condition that should be imposed is the requirement that $\phi \rightarrow \hat{\phi}$ at the boundary ∂dS . This boundary condition may be satisfied by identifying a boundary Green's function $K(\eta; k)$ that is a solution to the field equation (2) and obeys the condition $K \rightarrow 1$ on ∂dS [72]. In coordinate space, this is equivalent to the requirement that K approach a δ -function on the boundary and the appropriate solution is given by

$$K(\eta, k) = i \left(\frac{k}{2} \right)^{3/2} \Gamma(-1/2) (-\eta)^{3/2} H_{3/2}^{(2)}(-k\eta) = (1 - ik\eta) e^{ik\eta}. \quad (12)$$

The function K is sometimes referred to as the ‘bulk-to-boundary propagator’ and its momentum dependence is entirely specified by $k \equiv |\mathbf{k}|$.

The solution for the field ϕ which obeys the boundary condition (5) may now be written down immediately:

$$\phi(\eta, \mathbf{k}) = K(\eta, k) \hat{\phi}(\mathbf{k}). \quad (13)$$

The response $\bar{\phi}$ may be identified in terms of the source $\hat{\phi}$ by expanding solution (13) as an asymptotic series near $\eta \approx 0^-$ and comparing the coefficients of the expansion with the corresponding expansion of the general solution (9). More specifically, the asymptotic form of the solution (9) for a massless scalar field behaves quite generally near future infinity as

$$\phi \sim \hat{\phi} + \hat{\phi} \lambda \eta^2 + \bar{\phi} \eta^3 + \dots, \quad (14)$$

where ‘ \dots ’ denotes terms of $\mathcal{O}(\eta^4)$ or higher and λ is a constant, coming from the subleading term of the Δ_+ mode. It follows from Eq. (14) that the coefficient of the η^3 term in the near-boundary expansion of ϕ is indeed the response $\bar{\phi}$. Hence, expanding the solution (13) near the boundary, where $\eta \approx 0^-$, and evaluating the coefficient of the η^3 term, implies that the response is given by

$$\bar{\phi}(\mathbf{k}) = i \frac{k^3}{3} \hat{\phi}(\mathbf{k}). \quad (15)$$

3.2. Interacting Fields

This formalism can be extended to interacting fields. Within the context of the AdS/CFT correspondence, interactions were introduced in Ref. [50] and the first connected correlation functions containing three or more fields were derived in [73, 55, 54].

An interacting field implies that the potential V in Eq. (2) must contain cubic or higher-order couplings with some coupling constant, g . Our motivation for considering interacting fields arises from the possibility that primordial non-Gaussianities, which would be sourced by inflaton interactions, may soon be detectable in the CMB temperature anisotropy power spectrum. We will restrict our discussion to cubic

interactions, since these are of most relevance to inflationary cosmology. To be specific, we consider a bulk action of the form

$$S = \int d\eta d^3x \left[\frac{1}{2} a^2 (\phi'^2 - (\partial\phi)^2) + L_3 \right], \quad (16)$$

where L_3 denotes interaction terms that are of order ϕ^3 with coupling g . (These are given later by Eq. (55) for the inflationary scenario). Without loss of generality, we assume that no boundary terms are present in Eq. (16). If this is not the case, such terms can be removed, either by a field redefinition or by including a total derivative in the bulk interaction term L_3 .

The corresponding field equation for ϕ follows after varying Eq. (16):

$$\phi'' + 2\frac{a'}{a}\phi' - \partial^2\phi = \frac{1}{a^2} \frac{\delta L_3}{\delta\phi}, \quad (17)$$

where a prime denotes differentiation with respect to conformal time and $\delta L_3/\delta\phi$ is defined by the rule

$$\int d\eta d^3x \delta L_3 = \int d\eta d^3x \frac{\delta L_3}{\delta\phi} \delta\phi + \int_{\partial} d^3x \xi_2 \delta\phi, \quad (18)$$

in which the boundary term ξ_2 is generated by the integrations by parts that may be needed in order to cast δL_3 in the form $(\delta L_3/\delta\phi)\delta\phi$. In calculating the field equation, we are free to choose the boundary conditions for $\delta\phi$ so that it vanishes on the boundary. As a result, ξ_2 does not contribute to Eq. (17). However, as we shall see, this does *not* necessarily imply that such a term is irrelevant: although it has no consequences for the field equations, ξ_2 plays a vital role in determining the correct 1-point function, $\langle \mathcal{O} \rangle$. Our treatment improves the analysis of Mück & Viswanathan [50], who allowed for the possibility of arbitrary interactions in the AdS/CFT framework but took the boundary term described by ξ_2 to generate a contribution to the ϕ field equation (or implicitly allowed a modification to the action which would cancel any such terms). We discuss this point in more detail below.

The presence of interaction terms implies that the field equation, Eq (17), can not be solved exactly. As a result, we make the standard assumption that $|g| \ll 1$ and then perform a calculation in perturbation theory around g . Within the context of the inflationary scenario, g will be of order $\dot{\phi}/H$, and this assumption is equivalent to invoking the slow-roll approximation. We denote by a subscript ‘ n ’ terms that are of order g^n . Thus, at $O(g^0)$, Eq. (17) reduces to the free field case

$$\phi_0'' - \frac{2}{\eta}\phi_0' + k^2\phi_0 = 0 \quad (19)$$

and the solution to Eq. (19) that satisfies the Dirichlet boundary condition $\phi_0 \rightarrow \hat{\phi}$ on ∂dS is given by the boundary Green’s function (13) with ϕ replaced by ϕ_0 .

At the next order, Eq. (18) can be solved using the bulk Green’s function $G(\eta, \tau; k)$, which depends only on $k = |\mathbf{k}|$, and satisfies

$$\left(\frac{\partial^2}{\partial \eta^2} - \frac{2}{\eta} \frac{\partial}{\partial \eta} + k^2 \right) G(\eta, \tau; k) = \delta(\eta - \tau). \quad (20)$$

The boundary conditions for G should be consistent with the dS/CFT interpretation. Specifically, G should be regular in the deep interior of spacetime, where $\eta \rightarrow -\infty$. Near the boundary, $\eta \uparrow 0$, we require $G \rightarrow 0$, so that the $O(g^1)$ corrections to ϕ_0 do not violate the boundary condition $\phi \rightarrow \hat{\phi}$. The solution satisfying these conditions is

$$G(\eta, \tau; k) = \frac{\pi i}{2} (-\eta)^{3/2} (-\tau)^{-1/2} \times \begin{cases} H_{3/2}^{(2)}(-k\tau) J_{3/2}(-k\eta), & \eta > \tau \\ H_{3/2}^{(2)}(-k\eta) J_{3/2}(-k\tau), & \eta < \tau. \end{cases} \quad (21)$$

Hence, combining the $O(g^1)$ solution (21) with the $O(g^0)$ solution (12) yields the result

$$\phi(\eta, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^3} K(\eta; k) \hat{\phi}(\mathbf{k}) e^{-i\mathbf{k} \cdot \mathbf{x}} + \int \frac{d^3 k}{(2\pi)^3} d\tau G(\eta, \tau; k) \frac{1}{a^2} \widetilde{\frac{\delta L_3}{\delta \phi(\tau, \mathbf{k})}} e^{-i\mathbf{k} \cdot \mathbf{x}}, \quad (22)$$

where a tilde denotes the Fourier transform

$$\widetilde{\frac{\delta L_3}{\delta \phi(\tau, \mathbf{k})}} = \int d^3 x e^{i\mathbf{k} \cdot \mathbf{x}} \frac{\delta L_3}{\delta \phi(\tau, \mathbf{x})}, \quad (23)$$

and the τ integral in (22) is performed along a contour which is slightly rotated for large $|\eta|$, i.e., $\eta \mapsto \eta(1 - i\delta)$. In the remainder of this paper, integrals over conformal time such as Eq. (22) will be written merely as $\int_{-\infty}^0 d\tau$ with this convention understood. This rotation, which is already implicit in Eq. (16), is equivalent to the projection onto the interacting quantum vacuum [2] which appears in bulk QFT calculations of the three-point function [3, 74]. One might be tempted to imagine that this procedure, which is tantamount to dropping a rapidly oscillating term for $\eta \rightarrow -\infty$, corresponds to a renormalization prescription. However, this is not the case. Since it occurs in the field theory infra-red (equivalently, the gravitational ultra-violet) it has nothing to do with renormalization. The dangerous divergences which must be regulated all appear in the field theory ultra-violet (equivalently, the gravitational infra-red), where $\eta \approx 0^-$, and are safely removed by the holographic renormalization procedure to be discussed in Section 4.

It will be important in what follows that in the vicinity of the boundary, where $\eta \simeq 0^-$, the Green's function (21) behaves asymptotically as

$$G(\eta, \tau; k) \simeq \frac{\eta^3}{3\tau^2} K(\tau; k) + \dots, \quad (24)$$

where the dots denote higher-order terms in conformal time. This near-boundary behaviour of the Green's function can be employed to read off the coefficient of η^3 in the solution (22) for small η . As discussed above, this coefficient is the response $\bar{\phi}$. The contribution to the response from the bulk-to-boundary propagator in (22) is given by Eq. (15), whereas the contribution from the bulk-to-bulk propagator in the limit of small η is

$$\Delta \bar{\phi}(\eta, \mathbf{k}) \simeq \frac{\eta^3}{3} \int_{-\infty}^{\eta} \frac{d\tau}{a^2 \tau^2} K(\tau; k) \widetilde{\frac{\delta L_3}{\delta \phi(\tau, \mathbf{k})}} + \dots, \quad (25)$$

where we have written the integral explicitly only over the $\tau < \eta$ branch of (21). The other branch, denoted ‘ \dots ’, where $\tau > \eta$ and other higher-order terms in η contribute only at $O(\eta^4)$, and can be safely ignored for the purposes of calculating $\bar{\phi}$.

It follows, therefore, that the response for the interacting field is given by

$$\bar{\phi}(\mathbf{k}) = i\frac{k^3}{3}\hat{\phi}(\mathbf{k}) + \frac{H^2}{3}\int_{-\infty}^0 d\tau K(\tau; k)\widetilde{\frac{\delta L_3}{\delta\phi(\tau, \mathbf{k})}}, \quad (26)$$

where we have set the upper limit of integration to be zero rather than η , since the difference is of higher order in η and therefore irrelevant when calculating the response.

Note that the calculation in this section has been purely formal, without regard to the convergence of the τ integral in Eq. (26). In order to assign a precise meaning to this purely formal expression, one must carry out a renormalization procedure to remove possible infinities near the boundary surface $\eta \approx 0^-$. In the following Section, we proceed to calculate the one-point function of the deformed CFT in terms of the response (26), taking holographic renormalization into account.

4. The Holographic One-point Function

Eq. (7) cannot be precisely correct as it stands, since both sides are *a priori* divergent. On the gravitational side, the on-shell action $S_{\text{cl}}[\hat{\phi}]$ exhibits an infra-red divergence as $t \rightarrow \infty$. On the CFT side, one must expect the usual ultra-violet divergences of any local quantum field theory to be present. These divergences can be removed by the addition of appropriate counterterms, which should be understood to be included in Eq. (7).

In order to determine the nature of these counterterms, one regularizes the gravitational action by introducing a cut-off in the spacetime at some large, but finite, value of t , corresponding to a very small negative value of conformal time, $\eta = -\epsilon$, where $0 < \epsilon \ll 1$. We will denote the on-shell action computed in this regularized spacetime by S_ϵ . This action diverges as $\epsilon \rightarrow 0$, but after the divergences have been cancelled by the counterterms and the regularization removed, a finite contribution will remain. This remainder is interpreted as the renormalized CFT generating functional.

One might worry that this cut-off procedure is coordinate dependent, and that the results might change if the cut-off was calculated using a different choice of coordinate time. Our ability to reparametrize the spacetime cut-off corresponds to the insensitivity of the CFT to the regulator which is chosen. Moreover, although this subtraction scheme appears to explicitly break Lorentz covariance, it turns out that the counterterms can always be rewritten covariantly [52, 51]. Hence, if we calculate physical quantities with the specific regulator $\eta = -\epsilon$, the scheme-invariant quantities (such as observables of physical interest) should coincide with those calculated using any other regularization scheme.

4.1. The Holographic One-point Function in terms of the Response

It remains to explicitly calculate the one-point function $\langle \mathcal{O}(\mathbf{k}) \rangle_{\hat{\phi}}$. This will demonstrate the holographic renormalization procedure in action and allow us to determine the constant of proportionality in the relation $\langle \mathcal{O} \rangle \propto \bar{\phi}$. The argument is similar to Ref. [50], but care must be taken to include the effect of boundary terms.

We first integrate by parts in the quadratic sector of Eq. (16), after which one obtains

$$S_{\epsilon} = \int_{\eta=-\epsilon} d^3x \left[\frac{1}{2} a^2 \phi \phi' \right] - \int_{\eta=-\infty}^{\eta=-\epsilon} d\eta d^3x \left[\frac{1}{2} \phi (a^2 \phi')' - \frac{1}{2} a^2 \phi \partial^2 \phi \right] + \int_{\eta=-\infty}^{\eta=-\epsilon} d\eta d^3x L_3, \quad (27)$$

where the first term is evaluated on the slice corresponding to $\eta = -\epsilon$. It then follows from Eq. (17) that the on-shell action takes the form $S_{\epsilon} = S_{\epsilon|1} + S_{\epsilon|2} + S_{\epsilon|3}$, where

$$\begin{aligned} S_{\epsilon|1} &= \int_{\eta=-\epsilon} d^3x \left[\frac{1}{2} a^2 \phi \phi' \right] \\ S_{\epsilon|2} &= - \int_{\eta=-\infty}^{\eta=-\epsilon} d\eta d^3x \frac{1}{2} \phi \frac{\delta L_3}{\delta \phi} \\ S_{\epsilon|3} &= \int_{\eta=-\infty}^{\eta=-\epsilon} d\eta d^3x L_3. \end{aligned} \quad (28)$$

Eqs. (7) and (8) imply that the one-point function is determined by the variation $\delta S_{\epsilon} / \delta \hat{\phi}$ in the limit $\epsilon \rightarrow 0$, and this may be evaluated by considering the variation of each of the terms in Eq. (28) separately. The first term is a surface integral evaluated at small η and the expansion given in Eq. (14) implies that this can be expressed in the form

$$S_{\epsilon|1} \simeq \int_{\eta=-\epsilon} d^3x \left[-\frac{1}{\epsilon} \frac{\hat{\phi}^2 \lambda}{H^2} + \frac{3}{2} \frac{\hat{\phi} \bar{\phi}}{H^2} + \mathcal{O}(\epsilon) \right]. \quad (29)$$

The first term in Eq. (29) diverges as $\epsilon \rightarrow 0$ and should therefore be subtracted by an appropriate counterterm. The second term is finite, whereas the remaining terms are all $\mathcal{O}(\epsilon)$ and therefore vanish as the regulator is removed. After substitution of Eq. (26), therefore, this contribution to the on-shell action reduces to

$$\begin{aligned} S_{\epsilon|1}^{\text{ren}} &\simeq \int \frac{d^3k_1 d^3k_2}{(2\pi)^3} \delta(\mathbf{k}_1 + \mathbf{k}_2) \left[\frac{ik_1^3}{2H^2} \hat{\phi}(\mathbf{k}_1) \hat{\phi}(\mathbf{k}_2) \right. \\ &\quad \left. + \frac{1}{2} \int_{-\infty}^0 d\tau K(\tau; k_2) \hat{\phi}(\mathbf{k}_1) \widetilde{\frac{\delta L_3}{\delta \phi(\tau, \mathbf{k}_2)}} \right], \end{aligned} \quad (30)$$

where we write \simeq to denote expressions that are valid up to terms which vanish as $\epsilon \rightarrow 0$. Varying with respect to $\hat{\phi}(\mathbf{k})$ then implies that[§]

$$\frac{\delta S_{\epsilon|1}^{\text{ren}}}{\delta \hat{\phi}(\mathbf{k})} \simeq \frac{1}{(2\pi)^3} \frac{ik^3}{H^2} \hat{\phi}(-\mathbf{k}) + \frac{1}{2} \frac{1}{(2\pi)^3} \int_{-\infty}^0 d\tau K(\tau; k) \widetilde{\frac{\delta L_3}{\delta \phi(\tau, -\mathbf{k})}}$$

[§] A potentially subtle point is that no surface terms are ever generated from a variation with respect to the *boundary* field, i.e., variations of the form $\delta / \delta \hat{\phi}$, even when applied to derivatives of ϕ . For

$$+ \frac{1}{2} \int_{-\infty}^0 d\tau \frac{d^3 p}{(2\pi)^3} K(\tau; p) \hat{\phi}(\mathbf{p}) \frac{\delta}{\delta \hat{\phi}(\mathbf{k})} \frac{\widetilde{\delta L_3}}{\delta \phi(\tau, -\mathbf{p})}. \quad (32)$$

We also require the variations of the other terms in Eq. (28). After translating to Fourier space, the variation of $S_{\epsilon|2}$ is given by

$$\begin{aligned} \frac{\delta S_{\epsilon|2}}{\delta \hat{\phi}(\mathbf{k})} &= -\frac{1}{2} \frac{1}{(2\pi)^3} \int_{-\infty}^{-\epsilon} d\tau K(\tau; k) \frac{\widetilde{\delta L_3}}{\delta \phi(\tau, -\mathbf{k})} \\ &\quad - \frac{1}{2} \int_{-\infty}^{-\epsilon} d\tau \frac{d^3 p}{(2\pi)^3} K(\tau; p) \hat{\phi}(\mathbf{p}) \frac{\delta}{\delta \hat{\phi}(\mathbf{k})} \frac{\widetilde{\delta L_3}}{\delta \phi(\tau, -\mathbf{p})}. \end{aligned} \quad (33)$$

On the other hand, one can show by employing rule (18) (which expresses how δL_3 is related to $\delta L_3/\delta \phi$ and the surface term ξ_2) that $\delta S_{\epsilon|3}$ behaves like

$$\frac{\delta S_{\epsilon|3}}{\delta \hat{\phi}(\mathbf{k})} = \frac{1}{(2\pi)^3} \xi_2(-\epsilon, -\mathbf{k}) K(-\epsilon, k) + \int_{-\infty}^{-\epsilon} \frac{d\tau}{(2\pi)^3} K(\tau; k) \frac{\widetilde{\delta L_3}}{\delta \phi(\tau, -\mathbf{k})}. \quad (34)$$

Finally, after collecting together Eqs. (32), (33) and (34), we find that

$$\frac{\delta S_\epsilon}{\delta \hat{\phi}(\mathbf{k})} \simeq \frac{1}{(2\pi)^3} \frac{ik^3}{H^2} \hat{\phi}(-\mathbf{k}) + \int_{-\infty}^{-\epsilon} d\tau K(\tau, k) \frac{\widetilde{\delta L_3}}{\delta \phi(\tau, -\mathbf{k})} + \frac{1}{(2\pi)^3} \xi_2(-\epsilon, -\mathbf{k}) K(-\epsilon, k). \quad (35)$$

In order to take the $\epsilon \rightarrow 0$ limit, we define a renormalized response, $\bar{\phi}^{\text{ren}}$, such that

$$\bar{\phi}^{\text{ren}}(\mathbf{k}) = \lim_{\epsilon \rightarrow 0} \left(i \frac{k^3}{3} \hat{\phi}(\mathbf{k}) + \frac{H^2}{3} \int_{-\infty}^{-\epsilon} d\tau K(\tau, k) \frac{\widetilde{\delta L_3}}{\delta \phi(\tau, \mathbf{k})} + \text{counterterms} \right), \quad (36)$$

where the counterterms are chosen so that the limit exists [53]. The usual ambiguities arise when considering finite counterterms, which are independent of ϵ . These counterterms shift the final value of $\langle \mathcal{O}(\mathbf{k}) \rangle$, but since they are renormalization-scheme dependent, we can always choose a scheme in which they are absent. This is the “minimal” subtraction prescription of Ref. [53]. Applying the same procedure to ξ_2 yields a renormalized surface term:

$$\xi_2^{\text{ren}}(\mathbf{k}) = \lim_{\epsilon \rightarrow 0} (\xi_2(-\epsilon, \mathbf{k}) K(-\epsilon, k) + \text{counterterms}). \quad (37)$$

Hence, the correctly renormalized one-point function can be written in the simple and explicit form

$$\langle \mathcal{O}(\mathbf{k}) \rangle_{\hat{\phi}} = i \frac{\delta S_\epsilon}{\delta \hat{\phi}(\mathbf{k})} = \frac{1}{(2\pi)^3} \frac{3i}{H^2} \bar{\phi}^{\text{ren}}(-\mathbf{k}) + \frac{i}{(2\pi)^3} \xi_2^{\text{ren}}(-\mathbf{k}). \quad (38)$$

The presence of counterterms ensures that the integral in Eq. (36) is finite. This should be compared with the bare response, Eq. (26), which may contain divergences at future infinity. (In general, there are no divergences arising from the asymptotic past [31]).

example,

$$\frac{\delta}{\delta \hat{\phi}(\mathbf{k})} \frac{\partial^n}{\partial \eta^n} \phi(\eta, \mathbf{p}) = \frac{\delta}{\delta \hat{\phi}(\mathbf{k})} \frac{\partial^n}{\partial \eta^n} K(\eta; \mathbf{p}) \hat{\phi}(\mathbf{p}) = \frac{\partial^n}{\partial \eta^n} K(\eta; \mathbf{p}) \delta(\mathbf{p} - \mathbf{k}). \quad (31)$$

Surface terms only ever arise from variations with respect to the *bulk* field $\phi(\eta, \mathbf{x})$, rather than $\hat{\phi}(\mathbf{x})$.

Eqs. (36)–(37) can be used to obtain the holographic counterterms explicitly. However, such terms are always found to be imaginary in models that have been studied to date and Maldacena has conjectured that this property should hold in general [2]. In this case, it follows that since the interesting component of the one-point function is purely real, the correctly renormalized one-point function may be derived by dropping any terms in Eq. (35) which give rise to an imaginary part in $\langle \mathcal{O}(\mathbf{k}) \rangle$. This prescription is simpler to use in practice, although it should be emphasized that Eqs. (36)–(37) can always be employed to compute the counterterms without making any prior assumptions about their complex nature.

Eq. (38) differs from results previously obtained in the AdS/CFT context [50, 53, 51, 52] due to the presence of the surface contribution ξ_2 . This term arises because, as is usual in field theory in de Sitter space, we have discarded boundary terms at future infinity when calculating the field equations. This is the usual situation. In Ref. [50], Mück & Viswanathan assumed that the interaction term was written as I_{int} , which is equal to $\int d^4x L_3$ in our notation, and the field equations were taken to be $-\nabla^a \nabla_a \phi = \delta I_{\text{int}} / \delta \phi$. As a result, Mück & Viswanathan derived Eq. (38) with $\xi_2 = 0$. There is no discrepancy, because if L_3 does not contain derivative terms then ξ_2 is always zero, whereas if such terms are present then $\delta I_{\text{int}} / \delta \phi$ implicitly contains a δ -function term at the boundary which would replicate the effect of ξ_2 . As a result, our analysis is consistent with previous AdS/CFT computations which did not include derivative interactions [51, 52, 53, 60].

To summarize thus far, we have calculated the holographic one-point function $\langle \mathcal{O}(\mathbf{k}) \rangle_{\bar{\phi}}$ in terms of the response $\bar{\phi}$ and verified that after renormalization the two are directly proportional to one another, modulo a boundary term. Clearly, the specific functional form of the cubic interaction Lagrangian, $\delta L_3 / \delta \phi$, and the boundary term, ξ_2 , will depend on the nature of the effective field theory in question. However, before we proceed to discuss the effective action for perturbations in the inflaton field, we will first discuss an alternative parametrization for the one-point function that will prove useful in what follows.

4.2. The Holographic One-point Function in terms of the Interaction Lagrangian

Following [55], we may quite generally define an operator, \mathcal{X} , such that

$$\frac{\widetilde{\delta L_3}}{\delta \phi(\eta, \mathbf{k}_1)} = \int d^3x e^{i\mathbf{k}_1 \cdot \mathbf{x}} [\mathcal{X}(\mathbf{k}_1, \partial_2, \partial_3) \phi_0(\eta, \mathbf{x}_2) \phi_0(\eta, \mathbf{x}_3)]_{\mathbf{x}_2 = \mathbf{x}_3 = \mathbf{x}}, \quad (39)$$

where $\mathcal{X}(x, y, z)$ is a sum of powers of x , y and z and the derivatives ∂_1 and ∂_2 act only on \mathbf{x}_1 and \mathbf{x}_2 , respectively. The functional form of \mathcal{X} is determined by the nature of the cubic couplings in the interaction potential. We allow \mathcal{X} to contain a \mathbf{k}_1 dependence (where the subscript ‘1’ is introduced for future convenience) in order to accommodate interactions^{||} of the form $L_3 \propto \phi \partial^{-2} \phi \partial^2 \phi$ which, when written in terms of the field

^{||} The operator ∂^{-2} is the solution operator for the Laplacian, defined such that $\partial^{-2}(\partial^2 \phi) = \phi$.

equations, generates a source that includes terms of the form $\delta L_3/\delta\phi \propto \partial^2(\phi\partial^{-2}\phi)$. For such sources, \mathcal{X} can be written as the k_1 dependent expression $\mathcal{X} \propto -k_1^2\partial_3^{-2}$ (for example).

When L_3 represents a cubic interaction, standard manipulations imply that Eq. (39) can be written as a convolution over two copies of ϕ_0 such that

$$\begin{aligned} \frac{\widetilde{\delta L_3}}{\delta\phi(\eta, \mathbf{k}_1)} &= \int \frac{d^3k_2 d^3k_3}{(2\pi)^3} \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \mathcal{X}(\mathbf{k}_1, -\mathbf{k}_2, -\mathbf{k}_3) \phi_0(\eta, -\mathbf{k}_2) \phi_0(\eta, -\mathbf{k}_3) \\ &= \int \frac{d^3k_2 d^3k_3}{(2\pi)^3} \delta(\sum_i \mathbf{k}_i) \mathcal{X}_{123} K(\eta, k_2) K(\eta, k_3) \hat{\phi}(-\mathbf{k}_2) \hat{\phi}(-\mathbf{k}_3), \end{aligned} \quad (40)$$

where in the second expression we have written ϕ_0 in terms of the bulk-to-boundary propagator (13) and \mathcal{X}_{123} is a convenient shorthand for $\mathcal{X}_{123} = \mathcal{X}(\mathbf{k}_1, -\mathbf{k}_2, -\mathbf{k}_3)$. This is sufficient to parametrize the bare response, Eq. (26), in the form

$$\begin{aligned} \bar{\phi}^{\text{ren}}(\mathbf{k}_1) &= i\text{Im} \left(\frac{ik_1^3}{3} \hat{\phi}(\mathbf{k}_1) \right. \\ &\quad \left. + \frac{H^2}{3} \int d\tau \int \frac{d^3k_2 d^3k_3}{(2\pi)^3} \delta(\sum_i \mathbf{k}_i) \mathcal{X}_{123} K_1 K_2 K_3 \hat{\phi}(-\mathbf{k}_2) \hat{\phi}(-\mathbf{k}_3) \right), \end{aligned} \quad (41)$$

where $K_j \equiv K(\tau, k_j)$.

A similar parametrization may be employed for the (bare) boundary term, ξ_2 . Indeed, by following an identical argument to that which led to Eq. (40), we may write

$$\xi_2^{\text{ren}}(\eta, \mathbf{k}_1) = i\text{Im} \left(\lim_{\eta \rightarrow 0} \int \frac{d^3k_2 d^3k_3}{(2\pi)^3} \delta(\sum_i \mathbf{k}_i) \mathcal{Y}_{123} K(\eta, k_2) K(\eta, k_3) \hat{\phi}(-\mathbf{k}_2) \hat{\phi}(-\mathbf{k}_3) \right) \quad (42)$$

and Eq. (42) should be viewed as the definition of the quantity $\mathcal{Y}_{123} \equiv \mathcal{Y}(\mathbf{k}_1, -\mathbf{k}_2, -\mathbf{k}_3)$.

Hence, after substituting Eqs. (41) and (42) into Eq. (38) for the one-point function $\langle \mathcal{O}(\mathbf{k}) \rangle_{\hat{\phi}}$, we find that

$$\begin{aligned} \langle \mathcal{O}(\mathbf{k}_1) \rangle_{\hat{\phi}} &= \text{Re} \left(-\frac{1}{(2\pi)^3} \frac{k_1^3}{H^2} \hat{\phi}(-\mathbf{k}_1) \right. \\ &\quad + \frac{i}{(2\pi)^3} \int_{-\infty}^0 d\tau \int \frac{d^3k_2 d^3k_3}{(2\pi)^3} \delta(-\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \mathcal{X}_{123} K_1 K_2 K_3 \hat{\phi}(-\mathbf{k}_2) \hat{\phi}(-\mathbf{k}_3) \\ &\quad \left. + \frac{i}{(2\pi)^3} \lim_{\eta \rightarrow 0} \int \frac{d^3k_2 d^3k_3}{(2\pi)^3} \delta(-\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \mathcal{Y}_{123} K_1 K_2 K_3 \hat{\phi}(-\mathbf{k}_2) \hat{\phi}(-\mathbf{k}_3) \right). \end{aligned} \quad (43)$$

Now that the one-point function in the presence of the source $\hat{\phi}$ has been determined, all higher n -point functions may be derived from this quantity by functionally differentiating with respect to $\hat{\phi}$ and specifying $\hat{\phi} = 0$ afterwards. In the following Section, we present the general expressions for the two- and three-point functions.

5. Holographic Two- and Three-point Functions

5.1. Two-point Function

To obtain the two-point function, we substitute Eq. (26) into Eq. (38) and differentiate. It follows that [42]

$$\langle \mathcal{O}(\mathbf{k}_1) \mathcal{O}(\mathbf{k}_2) \rangle = \frac{\delta}{\delta \hat{\phi}(\mathbf{k}_2)} \langle \mathcal{O}(\mathbf{k}_1) \rangle_{\hat{\phi}} \Big|_{\hat{\phi}=0} = -\frac{1}{(2\pi)^3} \frac{k_1^3}{H^2} \delta(\mathbf{k}_1 + \mathbf{k}_2). \quad (44)$$

Note that there is no contribution to the two-point function from the surface term ξ_2 .

5.2. Three-point Function

The three-point function is obtained from the second functional derivative of Eq. (43):

$$\langle \mathcal{O}(\mathbf{k}_1) \mathcal{O}(\mathbf{k}_2) \mathcal{O}(\mathbf{k}_3) \rangle = \frac{\delta}{\delta \hat{\phi}(\mathbf{k}_2)} \frac{\delta}{\delta \hat{\phi}(\mathbf{k}_3)} \langle \mathcal{O}(\mathbf{k}_1) \rangle_{\hat{\phi}} \Big|_{\hat{\phi}=0}. \quad (45)$$

The presence of two functional derivatives means that even though we calculated to $\mathcal{O}(g)$ in the term that is quadratic in $\hat{\phi}$, it was not necessary to account for $\mathcal{O}(g)$ terms in the piece that is linear in $\hat{\phi}$. (This would have been necessary to self-consistently solve the field equations to $\mathcal{O}(g)$.) Such terms would correspond to loop corrections to the two-point function of the sort calculated in Ref. [31]. Hence, Eq. (45) yields the concise expression

$$\begin{aligned} \langle \mathcal{O}(\mathbf{k}_1) \mathcal{O}(\mathbf{k}_2) \mathcal{O}(\mathbf{k}_3) \rangle &= \text{Re} \frac{2i}{(2\pi)^6} \delta(\sum_i \mathbf{k}_i) \left[\int_{-\infty}^0 d\tau \text{sym}_{1,2,3} (\mathcal{X}_{123} K_1 K_2 K_3) \right. \\ &\quad \left. + \lim_{\eta \rightarrow 0} \int \text{sym}_{1,2,3} (\mathcal{Y}_{123} K_1 K_2 K_3) \right], \end{aligned} \quad (46)$$

where $\text{sym}_{1,2,3}$ denotes symmetrization with weight unity over the labels 1, 2 and 3. This symmetrization must occur since Eq. (46) was derived from Eq. (29) by three functional variations. In Ref. [55], this symmetrization was achieved in a different way, by symmetrization over $2 \leftrightarrow 3$ and integrating the result by parts to obtain symmetry over the labels 1, 2 and 3 in combination. However, the result must be the same.¶ Note also that the combination of signs for the various momenta \mathbf{k}_i which appear in Eq. (43) is necessary to obtain the correct expression for momentum conservation in Eq. (46). We have assumed that \mathcal{X}_{123} and \mathcal{Y}_{123} do not change under a change of sign $\mathbf{k}_i \mapsto -\mathbf{k}_i$. It is easy to verify that this is the case for the effective field theory of inflation, because \mathcal{X}_{123} and \mathcal{Y}_{123} depend on momentum only via the combinations ∂^2 and ∂^{-2} , which are

¶ There is a potential subtlety with this argument, because the one-point function is calculated from the regularized action, with a cut-off at $\eta = -\epsilon$. The cut-off is then removed, and higher correlation functions are obtained by functional differentiation. Thus, the two- and three-point functions are not precisely obtained by functional variation of a bosonic action, because of the intervening limiting process. This point was made explicitly in [55]. We assume this subtlety makes no difference to the correlation functions we are calculating.

invariant under sign exchange. In more general theories, however, sign changes in \mathbf{k}_i may introduce extra signs in (46), which should be correctly accounted for.

Thus far, the discussion has been entirely general, in the sense that we have not yet specified the precise form of the cubic interactions. In the following Section, we proceed to consider the case that is relevant to inflationary perturbation theory.

6. Effective Field Theory for the Inflaton

6.1. Third-order Action

We consider a universe sourced by a single scalar ‘inflaton’ field φ that is minimally coupled to Einstein gravity and self-interacting through a potential $W(\varphi)$. The four-dimensional bulk action is given by

$$S = \int d^4x \sqrt{-\det g} \left[\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - W(\varphi) \right]. \quad (47)$$

We assume that the background solution corresponds to a homogeneous scalar field $\varphi(\eta)$ propagating on the de Sitter spacetime. The evolution of fluctuations in the inflaton field may then be described by introducing small spatially dependent perturbations in the metric and the scalar field and expanding the action (47) in powers of these perturbations. It proves most convenient to work in the uniform curvature gauge and we denote the (gauge-invariant) scalar field perturbation by q . This can be related via a change of gauge to the comoving curvature perturbation \mathcal{R} or the uniform-density curvature perturbation ζ . The gravitational perturbations can then be directly related to q via constraint equations.

Expanding action (47) to third order in q and to leading order in the slow-roll parameter $\dot{\varphi}/H$ implies that the second- and third-order contributions to the action take the form [4, 2]

$$S_2 = \int d\eta d^3x \left[\frac{1}{2}a^2 q'^2 - \frac{1}{2}a^2 (\partial q)^2 \right] \quad (48)$$

$$S_3 = - \int d\eta d^3x \left[aq' \partial\psi \partial q + \frac{a^2 \dot{\varphi}}{4H} q q'^2 + \frac{a^2 \dot{\varphi}}{4H} q (\partial q)^2 \right] + \dots, \quad (49)$$

where ‘ \dots ’ denote terms which contain higher powers of q or $\dot{\varphi}/H$, and, as discussed after Eq. (45), we include only the leading order slow-roll piece at each order in q . The auxiliary function ψ is a gravitational perturbation and satisfies the constraint equation

$$\partial^2 \psi = -\frac{a\dot{\varphi}}{2H} q'. \quad (50)$$

Eq. (49) follows directly from Eq. (47) without performing any field redefinitions or integrations by parts in η , which would produce auxiliary boundary terms at $\eta = 0^-$. On the other hand we have freely integrated by parts in the spatial variables x^i which do not produce any surface terms.

After substituting for the gravitational perturbation, ψ , one finds that the cubic interaction terms in Eq. (49) can be rewritten in the form (see also [66])

$$S_3 = \int d\eta d^3x \left[-\frac{\dot{\phi}}{4H} \partial^{-2} \frac{\delta L}{\delta q} \Big|_1 (\partial q)^2 - \frac{a^2 \dot{\phi}}{2H} q' \partial^{-2} q' \partial^2 q - \frac{a^2 \dot{\phi}}{4H} q q'^2 \right] - \int_{\eta=0} d^3x \frac{a^2 \dot{\phi}}{4H} \partial^{-2} q' (\partial q)^2, \quad (51)$$

where $\delta L/\delta q|_1$ is the first-order equation of motion:

$$\frac{1}{a^2} \frac{\delta L}{\delta q} \Big|_1 = \frac{2}{\eta} q' - q'' + \partial^2 q. \quad (52)$$

Thus, $\delta L/\delta q|_1 = 0$ when evaluated on the classical solution.

The boundary term in Eq. (51) may be removed by performing a field redefinition of the form $q \mapsto \phi + F(q)$. This implies that the second-order action (48) transforms to

$$S_2[q] \mapsto S_2[\phi] + \int d\eta d^3x F(q) \frac{\delta L}{\delta q} \Big|_1 + \int_{\eta=0} d^3x a^2 q' F(q). \quad (53)$$

Hence, specifying F so that it satisfies the condition

$$F = \frac{\dot{\phi}}{4H} \partial^{-2} (\partial q)^2 \quad (54)$$

implies that the field redefinition $q \mapsto \phi + F(q)$ removes both the $\delta L/\delta q|_1$ term and the boundary integral \int_{∂} from the third-order action (51). A reduced interaction term of the form

$$S_3 = \int d\eta d^3x \left[-\frac{a^2 \dot{\phi}}{2H} \phi' \partial^{-2} \phi' \partial^2 \phi - \frac{a^2 \dot{\phi}}{4H} \phi \phi'^2 \right] \quad (55)$$

is all that remains.

6.2. Holographic Three-point Function for the Inflaton

We may now derive the (renormalized) holographic three-point function of the deformed Euclidean CFT that is dual to the inflaton. We begin by varying the interaction sector of the bulk action, Eq. (55), in order to determine the specific forms of the contributions \mathcal{X}_{123} and \mathcal{Y}_{123} that arise in the general expression for the three-point correlation function, Eq. (46). Variation of the action (55) results in both bulk and boundary contributions, as summarized in Eq. (18). After comparing the former to the general expression (40), we deduce an expression for \mathcal{X}_{123} of the form

$$\mathcal{X}_{123} = \frac{a^2 \dot{\phi}}{2H} \left[\left(\frac{1}{2} + \frac{k_3^2}{k_1^2} + \frac{k_3^2}{k_2^2} - \frac{k_1^2}{k_2^2} \right) \partial_2 \partial_3 + \left(\frac{k_3^2}{k_2^2} + \frac{k_3^2}{k_1^2} \right) \partial_2^2 + \partial_3^2 \right. \\ \left. + 2 \frac{a'}{a} \left(\frac{k_3^2}{k_2^2} + \frac{k_3^2}{k_1^2} \right) \partial_2 + 2 \frac{a'}{a} \partial_3 \right], \quad (56)$$

where ∂_n denotes an η -derivative which acts in Eq. (40) on the bulk-to-boundary propagator K_n , where K_n is given by Eq. (12).

Likewise, the form of the surface term is deduced by comparing the boundary terms that arise in the variation of the action (55) directly with Eq. (42). We find that

$$\mathcal{Y}_{123} = -\frac{a^2 \dot{\varphi}}{2H} \left[\left(\frac{k_3^2}{k_2^2} + \frac{k_3^2}{k_1^2} \right) \partial_2 + \partial_3 \right]. \quad (57)$$

It only remains to evaluate the integrals in the three-point function (46). Let us consider the first integral in this expression. For a bulk-to-boundary propagator of the form (12), we find after integration by parts that

$$\int_{-\infty}^0 d\tau \mathcal{X}_{123} K_1 K_2 K_3 = \int_{-\infty}^0 d\tau \bar{\mathcal{X}}_{123} K_1 K_2 K_3 - \int_{\eta=0} \mathcal{Y}_{123} K_1 K_2 K_3, \quad (58)$$

where $\bar{\mathcal{X}}$ is defined by

$$\bar{\mathcal{X}}_{123} = \frac{a^2 \dot{\varphi}}{2H} \left[\left(-\frac{1}{2} - \frac{k_1^2}{k_2^2} \right) \partial_2 \partial_3 - \partial_1 \partial_3 - \left(\frac{k_3^2}{k_2^2} + \frac{k_3^2}{k_1^2} \right) \partial_1 \partial_2 \right]. \quad (59)$$

Hence, the boundary term \mathcal{Y}_{123} in the three-point correlator (46) is *precisely canceled* by the second term in Eq. (58). This implies that the three-point correlator may be simplified to

$$\langle \mathcal{O}(\mathbf{k}_1) \mathcal{O}(\mathbf{k}_2) \mathcal{O}(\mathbf{k}_3) \rangle = \text{Re} \frac{2i}{(2\pi)^6} \delta(\sum_i \mathbf{k}_i) \int_{-\infty}^0 d\tau \text{sym}_{1,2,3} (\bar{\mathcal{X}}_{123} K_1 K_2 K_3). \quad (60)$$

In order to evaluate the integral in Eq. (60), we employ the result:

$$i \int_{-\infty}^0 \frac{d\eta}{\eta^2} \partial_2 \partial_3 K_1 K_2 K_3 = \frac{k_2^2 k_3^2}{k_t} + \frac{k_1 k_2^2 k_3^2}{k_t^2}, \quad (61)$$

where Eq. (12) has been employed once more and $k_t \equiv k_1 + k_2 + k_3$. It follows, therefore, that

$$i \int_{-\infty}^0 d\eta \text{sym}_{1,2,3} [\bar{\mathcal{X}}_{123} K_1 K_2 K_3] = -\frac{\dot{\varphi}}{H^3} \frac{1}{k_t} \sum_{i < j} k_i^2 k_j^2 \quad (62)$$

and this implies that the three-point function may be expressed in the succinct form

$$\langle \mathcal{O}(\mathbf{k}_1) \mathcal{O}(\mathbf{k}_2) \mathcal{O}(\mathbf{k}_3) \rangle = -\frac{1}{(2\pi)^6} \delta(\sum_i \mathbf{k}_i) \frac{2\dot{\varphi}}{H^3 k_t} \sum_{i < j} k_i^2 k_j^2. \quad (63)$$

7. The Inflaton Three-point Function

The dictionary that relates the correlators for the bulk inflaton field to those for the dual CFT as calculated above was presented in Refs. [2, 38]. In particular, the two-point functions are related by

$$\langle \phi(\mathbf{k}_1) \phi(\mathbf{k}_2) \rangle = -\frac{1}{2\text{Re} \langle \mathcal{O}(\mathbf{k}_1) \mathcal{O}(\mathbf{k}_2) \rangle}, \quad (64)$$

and the analogous expression for the three-point functions is given by

$$\langle \phi(\mathbf{k}_1) \phi(\mathbf{k}_2) \phi(\mathbf{k}_3) \rangle = \frac{2\text{Re} \langle \mathcal{O}(\mathbf{k}_1) \mathcal{O}(\mathbf{k}_2) \mathcal{O}(\mathbf{k}_3) \rangle}{\prod_i (-2\text{Re} \langle \mathcal{O}(\mathbf{k}_i) \mathcal{O}(-\mathbf{k}_i) \rangle')}, \quad (65)$$

where $\langle \mathcal{OO} \rangle'$ represents the correlator with the momentum-conservation δ -function omitted.

Comparison between Eqs. (44) and (64) immediately yields the two-point function for the inflaton field [38]:

$$\langle \phi(\mathbf{k}_1) \phi(\mathbf{k}_2) \rangle = (2\pi)^3 \frac{H^2}{2k_1^3} \delta(\mathbf{k}_1 + \mathbf{k}_2). \quad (66)$$

The three-point function follows after substitution of Eqs. (63) and (66) into Eq. (65):

$$\langle \phi(\mathbf{k}_1) \phi(\mathbf{k}_2) \phi(\mathbf{k}_3) \rangle = -(2\pi)^3 \delta\left(\sum_i \mathbf{k}_i\right) \frac{4\dot{\phi}H^3}{\prod_i 2k_i^3} \frac{1}{k_t} \sum_{i<j} k_i^2 k_j^2. \quad (67)$$

The corresponding three-point function for the inflaton field perturbation, q , can then be determined by introducing the field redefinition (54) back into Eq. (67) and using Wick's theorem, as described in [2, 4]. This yields the final result:

$$\begin{aligned} \langle q(\mathbf{k}_1) q(\mathbf{k}_2) q(\mathbf{k}_3) \rangle &= (2\pi)^3 \delta\left(\sum_i \mathbf{k}_i\right) \frac{1}{\prod_i 2k_i^3} \dot{\phi} H^3 \\ &\times \left[\frac{1}{2} \sum_i k_i^3 - \frac{4}{k_t} \sum_{i<j} k_i^2 k_j^2 - \frac{1}{2} \sum_{i \neq j} k_i k_j^2 \right]. \end{aligned} \quad (68)$$

Eq. (68) is expression (68) of Ref. [4] specialized to a single field, which was derived from the bulk QFT calculation.

However, it is not necessary to perform a field redefinition in order to take into account the boundary terms of the form $\int_{\eta=0}$ in the third-order action (51). The three-point correlator (68) may also be derived by including the boundary term explicitly. The form of this term is given by

$$- \int_{\eta=0} d^3x \frac{a^2 \dot{\phi}}{4H} \partial^{-2} q' (\partial q)^2. \quad (69)$$

This is manifestly divergent at future infinity, because the scale factor a is unbounded at late times. This divergence is subtracted using our renormalization prescription, Eq. (69), and one may verify that it is purely imaginary. Furthermore, the oscillatory nature of the wavefunction at past infinity implies that there will be no contribution from regions where a given k -mode is deep inside the horizon [2, 31].

The boundary integral (69) may be evaluated when the Bunch-Davies vacuum is invoked as the initial state for the perturbation. In this case, the perturbation evolves as

$$q = \frac{(2\pi)^{3/2}}{\sqrt{2k^3}} H (1 - ik\eta) e^{ik\eta}, \quad (70)$$

which implies that, after renormalization, the boundary term yields a contribution to the q -correlator of the form

$$\Delta \langle q(\mathbf{k}_1) q(\mathbf{k}_2) q(\mathbf{k}_3) \rangle = (2\pi)^3 \delta\left(\sum_i \mathbf{k}_i\right) \frac{H^3 \dot{\phi}}{\prod_i 2k_i^3} \frac{\mathbf{k}_2 \cdot \mathbf{k}_3}{4} k_1 + \text{perms} + \text{c.c.}, \quad (71)$$

where c.c. denotes the complex conjugate. Finally, after taking the permutations into account and using the relation $\sum_i \mathbf{k}_i = 0$, we find that the contribution of the boundary term is

$$\Delta \langle q(\mathbf{k}_1) q(\mathbf{k}_2) q(\mathbf{k}_3) \rangle = (2\pi)^3 \delta\left(\sum_i \mathbf{k}_i\right) \frac{4\pi^4}{\prod_i k_i^3} \left(\frac{H}{2\pi}\right)^4 \frac{\dot{\varphi}}{4H} \left(\sum_i k_i^3 - \sum_{i \neq j} k_i k_j^2\right). \quad (72)$$

Combining Eqs. (67) and (72) therefore exactly reproduces the three-point correlator for the inflaton perturbation, Eq. (68).

8. Discussion

In this paper, we have demonstrated explicitly how the formalism of the dS/CFT correspondence may be employed to derive the primordial three-point correlation function of the inflaton field perturbation by calculating the bulk prediction for the corresponding three-point CFT correlator. This complements the standard bulk QFT approach based on evaluating tree-level Feynman diagrams. It also provides an important consistency check of Maldacena's formula [2, 38] relating the bulk and boundary correlators. We have also emphasized that the inflaton three-point function can be determined directly from the third-order action (51) without the need for a field redefinition if the contribution from the boundary integral is included. Although, for simplicity, we have limited the analysis to a single field, the extension to multiple-field models is straightforward.

When derivative interactions are present in the action, boundary terms at future infinity arise after varying the action to obtain the field equations. These terms have not previously been considered explicitly in the AdS/CFT case. In the analysis of Ref. [50], such interactions may implicitly be present, but the associated boundary terms are included in the field equations or are taken to vanish, which could be achieved either by picking appropriate boundary conditions for the fields, or by modifying the action. As is usual in de Sitter field theory, we have discarded the contribution of these terms from the bulk field equation. This has important consequences for the one-point correlation function of the dual CFT. We have shown explicitly in Eq. (38) that the one-point function in the dS/CFT correspondence is indeed proportional to the response, as given by Eq. (26), but only modulo a boundary term. This term contributes to the three-point function, as seen in Eq. (46). However, such a contribution is cancelled after the term that contains the variation of the cubic interaction Lagrangian is integrated by parts to remove higher-derivative operators. Hence, the three-point function can be calculated even if the form of the boundary contribution is unknown.

One of the attractive features of the method we have outlined above is that the holographic one-point function is determined by the variation of the interaction Lagrangian. This suggests that it should be possible in principle to determine the correlator from the *second-order* field equation and, furthermore, indicates that the n -point function of the primordial inflaton perturbation could be determined by employing

only $(n-1)^{\text{th}}$ -order perturbation theory. This is in contrast to conventional approaches, where the action must be evaluated to $\mathcal{O}(q^n)$ in order to obtain the n -point q -correlator.

To calculate the n -point function in this way, one would begin with the full action for Einstein gravity coupled to a scalar field (c.f. [12]), φ , which can be written in the form [4]

$$S = -\frac{1}{2} \int N \sqrt{h} [\nabla^i \varphi \nabla_i \varphi + 2W(\varphi)] + \frac{1}{2} \int \frac{\sqrt{h}}{N} [E^{ij} E_{ij} - E^2 + \pi^2], \quad (73)$$

where we have adopted the ADM line element,

$$ds^2 = -N^2 dt^2 + h_{ij} (dx^i + N^i) (dx^j + N^j) \quad (74)$$

with lapse function N and shift vector N^j , and π is the scalar field momentum, $\pi = \dot{\varphi} - N^j \nabla_j \varphi$. The spatial tensor E_{ij} is related to the extrinsic curvature of the spatial slices such that $E_{ij} = \frac{1}{2} \dot{h}_{ij} - \nabla_{(i} N_{j)}$. Eq. (73) should be evaluated by selecting a spatial gauge for h_{ij} , such as the comoving gauge where $h_{ij} = a^2(t) \delta_{ij}$, and parametrizing the inflaton field in terms of a homogeneous background component, φ_h , and a perturbation q , i.e., $\varphi = \varphi_h + q$.

The equations of motion for the fields N and N_j are given by

$$-\nabla^i \varphi \nabla_i \varphi - 2W(\varphi) - \frac{1}{N^2} (E^{ij} E_{ij} - E^2 + \pi^2) = 0 \quad (75)$$

and

$$\nabla_j \left(\frac{1}{N} [E_i^j - E \delta_i^j] \right) = \frac{1}{N} \pi \nabla_i \varphi, \quad (76)$$

respectively, and their solution specifies $N = N(\varphi_h, q)$ and $N_j = N_j(\varphi_h, q)$ as functions of the background inflaton trajectory and the perturbation q . The field equation for the inflaton fluctuation is also required and this follows by varying the action (73) with respect to q . This field equation can then be used to determine the quantities \mathcal{X}_{123} and \mathcal{Y}_{123} , from which the renormalized response $\bar{\phi}^{\text{ren}}$ and boundary term ξ_2^{ren} follow, thereby yielding the correlator.

In general, however, the obstacle to this procedure is that the solution of the constraint equations for the lapse and shift is a complicated function of the inflaton perturbation q . Consequently, when determining the three-point correlator, it is as straightforward to reduce the action (73) to a functional in q^3 and calculate the correlator using Feynman graphs, as it is to use the field equation in the holographic approach. On the other hand, for 4- and higher n -point functions, the prospect of reducing the order of perturbation theory by one may provide a significant technical simplification. This will become more relevant in the future as the quality of CMB data improves. Indeed, constraints on inflationary non-Gaussianity based on the primordial trispectrum have recently been discussed [20, 29, 75] and it is of importance to explore these issues further.

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